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Theory of ion transport in the tokamak SOL

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Abstract

Edge plasmas, such as the tokamak scrape-off layer, exist as a consequence of a balance between cross field diffusion and parallel losses. This circumstance is shown to lead to an unconventional form of the parallel ion transport laws under certain conditions. The cross field diffusion, whether it is classical or anomalous, affects the parallel ion transport by modifying the parallel friction force between different ion species, which could be important for impurity retention in a tokamak divertor.

Keywords: Impurity transport; Kinetic analysis; Turbulence

1. Introduction

Numerical models of edge plasmas usually employ fluid equations of the type derived e.g. by Braginskii [1] for a pure hydrogen plasma and later generalized to describe impure plasmas with an arbitrary number of ion species [2–4]. However, these equations were derived under assumptions that are not always true in the tokamak edge. Edge plasmas, such as the tokamak scrape-off layer (SOL), exist as a consequence of the balance between cross field diffusion and parallel losses. The conventional fluid equations do not, in principle, allow for such a balance. To make this clear, let us consider the parallel component of the ion momentum equation in steady state:

$$m_j n_j (V_j \cdot \nabla) V_{j\|} = n_j e_j E_\| - \nabla_\| p_j - (\nabla \cdot \Pi_j)_\| + R_j, \quad (1)$$

where $E_\|$ is the parallel electric field, m_j is the mass, n_j the density, e_j the charge, $p_j = n_j T_j$ the pressure, Π_j the viscosity, V_j the flow velocity and R_j is the parallel force acting on ion species j as a consequence of collisions with the other species. If, for instance, the plasma consists of two ion species, hydrogenic ions, $j = i$ and heavy impurities, $j = z$, the force on the former is equal to [3]:

$$R_i = -C_1 \frac{m_i n_i (V_{i\|} - V_{z\|})}{\tau_{iz}} - C_2 n_i \nabla_\| T_i, \quad (2)$$

where $\tau_{iz} = 3 m_i^{1/2} T_i^{3/2} / 4(2\pi)^{1/2} n_z e_z^2 e_i^2 \ln \Lambda$ is the ion-impurity collision time, and C_1 and C_2 are coefficients depending on the impurity strength $\alpha = n_z e_z^2 / n_i e_i^2$, tabulated in Ref. [3]. Solving Eq. (1) for the parallel flow of impurities relative to the main ions gives:

$$V_{z\|} - V_{i\|} = -\frac{\tau_{iz}}{C_1 m_i n_i} \Big[-\nabla_\| p_i + n_i e_i E_\| - C_2 n_i \nabla_\| T_i - (\nabla \cdot \Pi_i)_\| - m_i n_i (V_i \cdot \nabla) V_{i\|} \Big]. \quad (3)$$

In this expression, the first three terms on the right-hand side are the principal parallel driving forces. The last two terms, which contain cross field gradients, are assumed to be small in the usual derivation of the fluid equations. It is thus implicitly assumed that parallel transport is driven primarily by parallel gradients. This is, however, not necessarily true in the SOL. It is a basic property of edge plasmas that transport does not occur within each flux tube separately since radial diffusion feeds in plasma from the core. In principle, it should even be possible for a parallel

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flow to be driven entirely by radial diffusion in the absence of parallel gradients.

In order to let the last two terms in Eq. (3) be as large as the the other terms, we must make the ordering [5]:

$$v_T/L_\parallel \sim D/W^2, \tag{4}$$

where v_T is the thermal speed, $L_\parallel$ the parallel scale length, W the SOL width (which is much smaller than the minor radius a) and D the radial diffusion coefficient. The latter is responsible for cross field drives in Eq. (3), giving:

$$(\nabla\cdot \Pi_i)_\parallel \sim -\frac{\partial}{\partial r} D m_i n_i \frac{\partial V_{i\parallel}}{\partial r} \sim m_i n_i D v_T/W^2 \tag{5}$$

$$m_i n_i (V_i\cdot\nabla)V_{i\parallel} \sim -m_i D\frac{\partial n_i}{\partial r}\frac{\partial V_{i\parallel}}{\partial r} \sim m_i n_i D v_T/W^2 \tag{6}$$

where we have taken the strongest gradients to be in the radial (r) direction and the radial velocity to be diffusive, $V_\perp\cdot\nabla \sim -D(\partial \ln n/\partial r)\partial/\partial r \sim D/W^2$. The parallel flow velocity $V_{i\parallel}$ is assumed to be of the order of the sound speed, as dictated by the Bohm sheath criterion.

The ordering Eq. (4) implies a balance between radial diffusion and parallel losses at sonic flows. In the present paper, novel kinetic equations are derived and solved, allowing for such a balance and transport laws are constructed for impure, collisional edge plasmas in which the perpendicular transport is either due to Coulomb collisions between different ion species, or governed by anomalous diffusion driven by electrostatic turbulence.

2. Classical diffusion

In this section we give an outline of the derivation [6] of ion transport laws with the ordering Eq. (4) when the diffusion is taken to be classical, caused by ion-impurity collisions. The plasma is assumed to consist of a single light main ion species, and heavy impurity ions. We are interested in magnetized SOLs so W remains larger than the ion gyroradius, ρ_i, resulting in three spatial scale lengths:

$$\rho_i \ll W \ll a, \tag{7}$$

where we take parallel and poloidal scale lengths to be comparable and of order a. The first of these inequalities is used to define the basic small parameter, $\delta \equiv \rho_i/W \ll 1$, characterizing a magnetized SOL. The second ratio, W/a, is treated as an independent small parameter. It is convenient to express the classical diffusion coefficient D_c in the form

$$D_c \equiv \rho_i^2/2\tau_{iz}, \tag{8}$$

with $\rho_i = v_T/\Omega_i$, and $\Omega_i = e_i B/m_i$. Then the basic ordering Eq. (4) describing the edge becomes:

$$\omega_t \sim \nu_{iz}\delta^2, \tag{9}$$

where $\nu_{iz} = 3\pi^{1/2}/8\tau_{iz}$ and where the motion within a flux surface is measured by the conventional transit frequency, $\omega_t = v_T/a$.

We employ a large drift velocity form of the drift kinetic equation [8]. To write the Lorentz operator for ion-impurity collisions in the simplest possible form, the frame velocity V is chosen to be that of the impurities, with $V_\parallel \sim v_T \gg V_\perp$. Ion–ion collisions are neglected for simplicity by assuming $n_z e_z^2 \gg n_i e_i^2$ so that the standard results can be recovered from the electron–ion Lorentz gas ones by replacing the electrons (ions) by the ions (impurities).

Using γ and $\epsilon = (v - V)^2/2$ to denote gyrophase and energy and letting $v - V = s + u$ with $s = s(e_2\cos\gamma - e_3\sin\gamma)$, $u = nn\cdot(v - V) = un$, $n = B/B$ and $n = e_2\times e_3$, the equation for the gyroaveraged ion distribution function $\bar f = \langle f\rangle$, for the edge orderings, can be shown to be:

$$(u+V)\cdot\nabla\bar f_i + \bar{\dot\epsilon}\frac{\partial \bar f_i}{\partial\epsilon} + \left\langle s\cos\gamma\left(\frac{\partial\tilde f_i}{\partial r} - u\frac{V_\parallel}{\partial r}\frac{\partial\tilde f_i}{\partial\epsilon}\right)\right\rangle = \langle C_i(f_i)\rangle + O(\rho_i\omega_t/W), \tag{10}$$

where in this section $\langle\ldots\rangle$ denotes gyroaverage, $\bar f_i = f_{Mi} = n_i(m_i/2\pi T)^{3/2}\exp(-m_i\epsilon/T)$ to lowest order and we neglect magnetic drifts (and, therefore, gyroviscosity) by assuming $\nu_{iz}\delta \gg \omega_t$. The gyrophase dependent distribution function $\tilde f_i = f_i - \bar f_i$ satisfies

$$\Omega_i\frac{\partial\tilde f_i}{\partial\gamma} = -s\cos\gamma\left(\frac{\partial f_{Mi}}{\partial r} - u\frac{\partial V_\parallel}{\partial r}\frac{\partial f_{Mi}}{\partial\epsilon}\right) + C_i(\tilde f_i) + \ldots, \tag{11}$$

to the requisite order. Solving iteratively for the leading $\cos\gamma$ term in $\tilde f_i$ by assuming $\nu_{iz} \ll \Omega_i$ and inserting the result in Eq. (10), gives the desired edge-ordered kinetic equation in which parallel streaming and radial diffusion enter at the same order:

$$(u+V)\cdot\nabla f_{Mi} + \langle\dot\epsilon\rangle\frac{\partial f_{Mi}}{\partial\epsilon} = C_i(f_i - f_{Mi}) + D(f_{Mi}), \tag{12}$$

with $\xi = u/(2\epsilon)^{1/2}$, $W_\parallel = n\cdot\nabla V\cdot n - \nabla\cdot V/3$, $E'_\parallel = -\nabla_\parallel\Phi - (m_i/e_i)(V\cdot\nabla V)_\parallel$,

$$D(f_{Mi}) = \left(\frac{\partial}{\partial r} - u\frac{\partial V_\parallel}{\partial r}\frac{\partial}{\partial\epsilon}\right)\left\{\frac{\nu_{iz}v_T^3 s^2}{\Omega_i^2(2\epsilon)^{3/2}}\left[\left(\frac{\partial f_{Mi}}{\partial r}\right)_\epsilon + \frac{6u}{v_T^2}\frac{\partial V_\parallel}{\partial r}\right]\right\}, \tag{13}$$

and

$$\langle\dot\epsilon\rangle = -\tfrac{2}{3}\epsilon\nabla\cdot V + \frac{e_i(2\epsilon)^{1/2}}{m_i}E'_\parallel\xi + \epsilon W_\parallel(1 - 3\xi^2). \tag{14}$$

From Eq. (12) conservation laws for particles, parallel momentum and energy can be derived. To the lowest-order they may be written as:

$$\nabla \cdot (\boldsymbol{n} n_i V_\parallel) - \frac{\partial}{\partial r}\left[D_c\left(\frac{\partial n_i}{\partial r} - \frac{n_i}{2T}\frac{\partial T}{\partial r}\right)\right] = 0, \tag{15}$$

$$\nabla_\parallel p_i - n_i e E_\parallel + m_i n_i V_\parallel \nabla_\parallel V_\parallel = R_i$$
$$+ \frac{\partial}{\partial r}\left(\tfrac{6}{5} m_i D_c n_i \frac{\partial V_\parallel}{\partial r}\right) + m_i D_c \frac{\partial V_\parallel}{\partial r}\left(\frac{\partial n_i}{\partial r} - \frac{n_i}{2T}\frac{\partial T}{\partial r}\right), \tag{16}$$

$$\nabla \cdot \left(\boldsymbol{n}\frac{3p_i}{2}\right) + p_i \nabla \cdot (\boldsymbol{n} V_\parallel) = \tfrac{6}{5} m_i n_i D_c \left(\frac{\partial V_\parallel}{\partial r}\right)^2$$
$$+ \frac{\partial}{\partial r}\left[D_c T\left(\frac{\partial n_i}{\partial r} + \frac{n_i}{2T}\frac{\partial T}{\partial r}\right)\right]. \tag{17}$$

When the higher-order moment equations are formed, additional moments of f_i enter, so that Eq. (12) must be solved for the correction to the Maxwellian by expanding in Legendre polynomials, solving a sequence of Spitzer [9] problems and integrating to evaluate the transport coefficients. Of particular interest is the relative ion-impurity flow:

$$V_{i\parallel} - V_{z\parallel} = -\frac{4}{\pi^{1/2}}\frac{T}{m_i \nu_{iz}}\left(\nabla_\parallel n_i - \frac{e_i}{T}E'_\parallel + \tfrac{5}{2}\nabla_\parallel T\right)$$
$$+ \frac{9\pi^{1/2}}{8\nu_{iz}}\frac{\partial}{\partial r}\left(n_i D_c \frac{\partial V_\parallel}{\partial r}\right) \tag{18}$$
$$+ \frac{3\pi^{1/2}}{5}\frac{D_c}{\nu_{iz}}\frac{\partial V_\parallel}{\partial r}\left(\frac{\partial n_i}{\partial r} + \frac{61 n_i}{16T}\frac{\partial T}{\partial r}\right). \tag{19}$$

Note that all of the terms are the same order. The last term involves products of radial gradients and is not part of the viscosity term that precedes it. Eq. (19) can be used to find the modification R_i^{edge} to the friction by writing $R_i = R_i^{\text{Brag}} + R_i^{\text{edge}}$, using the full parallel ion momentum balance equation and recalling that the usual result from Braginskii [1] is:

$$R_i^{\text{Brag}} = -\tfrac{3}{2} n_i \nabla_\parallel T - \frac{\pi^{1/2}}{4} m_i n_i \nu_{iz}(V_{i\parallel} - V_{z\parallel}). \tag{20}$$

The result is:

$$R_i^{\text{edge}} = m_i n_i D_c \frac{\partial V_\parallel}{\partial r}\left[\left(\frac{3\pi}{20} - 1\right)\frac{\partial \ln n_i}{\partial r}\right.$$
$$\left. + \left(\frac{1}{2} + \frac{183\pi}{320}\right)\frac{\partial \ln T}{\partial r}\right] \tag{21}$$
$$-\left(\frac{6}{5} - \frac{9\pi}{32}\right)\frac{\partial}{\partial r}\left(m_i n_i D_c \frac{\partial V_\parallel}{\partial r}\right), \tag{22}$$

which is not obtained in the standard orders and modifies the usual $R_i = 0$ result which tends to drive impurities towards higher temperature regions.

3. Anomalous diffusion

We now turn our attention to the case when the cross field transport is anomalous and driven by electrostatic turbulence [7]. In numerical modeling of the tokamak edge, anomalous diffusion coefficients across the magnetic field are usually invoked in such a way as to match experimentally observed density and temperature profiles. On the other hand, the transport is generally taken to be classical along the field. This ignores the strong coupling that exists between the transport along and across the field. As we shall see, if the radial transport is anomalous, parallel ion transport cannot in general be entirely classical.

We consider a turbulent, impure edge plasma with an arbitrary number of ion species present. The perpendicular wavelength, $2\pi/k_\perp$, of the turbulence is assumed to be large compared with the ion Larmor radius, $k_\perp \rho_i \ll 1$. Writing the $\boldsymbol{E} \times \boldsymbol{B}$ drift velocity, $\boldsymbol{V}_E = \bar{\boldsymbol{V}}_E + \tilde{\boldsymbol{V}}_E$ and the distribution function $f_i = \bar{f}_i + \tilde{f}_i$ for each ion species i, as sums of average and fluctuating parts and taking the average of the drift kinetic equation over fluctuations, gives:

$$(\boldsymbol{v}_\parallel + \boldsymbol{V}_d + \bar{\boldsymbol{V}}_E) \cdot \nabla \bar{f}_i + \langle \tilde{\boldsymbol{V}}_E \cdot \nabla \tilde{f}_i \rangle + \frac{e_i E_\parallel}{m_i}\frac{\partial \bar{f}_i}{\partial v_\parallel} = \bar{C}_i, \tag{23}$$

where we now use $\langle \ldots \rangle$ to denote the fluctuation average. $\bar{C}_i$ is the averaged collision operator with all other species and magnetic drifts have been neglected as ρ_i/W smaller than parallel streaming.

The anomalous diffusion generally invoked to account for the radial transport comes from the term $\langle \tilde{\boldsymbol{V}}_E \cdot \nabla \tilde{f}_i \rangle$ in Eq. (23). Even without employing any specific model for the radial transport, we note that it will, in general, affect the parallel transport whenever ordering Eq. (4) is satisfied. To demonstrate this phenomenon explicitly in the simplest possible situation, we take the turbulence to be weak. With the ordering $k_\perp \rho_i \sim k_\parallel v_T/\omega \sim (k_\perp W)^{-2} \sim \tilde{f}_i/\bar{f}_i \ll 1$, it can be argued [7] that the anomalous transport is purely diffusive:

$$\langle \tilde{\boldsymbol{V}}_E \cdot \nabla \tilde{f}_i \rangle = -\frac{\partial}{\partial r}\left(D_a \frac{\partial \bar{f}_i}{\partial r}\right), \tag{24}$$

with the anomalous diffusion coefficient:

$$D_a \equiv \frac{1}{2}\int_0^\infty \langle (\tilde{V}_{Er}(t)\tilde{V}_{Er}(0)\rangle \, \mathrm{d}t, \tag{25}$$

independent of the particle velocity. The integral in Eq. (25) is taken along particle orbits and converges in a few correlation times of the random $E \times B$ field. The expression Eq. (25) is quite general and can be expected to hold also for strong turbulence [10], provided classical diffusion is negligible, $k_\perp v_T \ll \omega$, and $k_\perp W \gg 1$.

We consider the short-parallel-mean-free-path limit of the drift kinetic Eq. (23) with Eq. (24) for the anomalous diffusion. The collision operator then dominates and to the lowest order the distribution function for each ion species becomes Maxwellian:

$$\bar{f}_{i0} = n_i \left(\frac{m_i}{2\pi T_i} \right)^{3/2} \exp\left(-\frac{m_i v_\perp^2}{2T_i} - \frac{m_i (v_\parallel - V_\parallel)^2}{2T_i} \right), \tag{26}$$

with a common parallel velocity $V_\parallel \sim v_T$. The temperatures T_i may be different for different species with very disparate masses m_i.

The moment equations associated with Eqs. (23), (24) and (26) are:

$$\nabla_\parallel (n_i V_\parallel) + \frac{\partial}{\partial r} \left(\bar{V}_{Er} n_i - D_a \frac{\partial n_i}{\partial r} \right) = 0, \tag{27}$$

$$\nabla_\parallel (p_i + m_i n_i V_\parallel^2) + \frac{\partial}{\partial r} \left[\left(\bar{V}_{Er} - D_a \frac{\partial}{\partial r} \right) m_i n_i V_\parallel \right] = R_i + n_i e_i E_\parallel, \tag{28}$$

$$\nabla_\parallel \left(\frac{5 n_i T_i V_\parallel}{2} + \frac{m_i n_i V_\parallel^3}{2} \right) + \frac{\partial}{\partial r} \left[\left(\bar{V}_{Er} - D_a \frac{\partial}{\partial r} \right) \left(\frac{3 n_i T_i}{2} + \frac{m_i n_i V_\parallel^3}{2} \right) \right] = n_i e_i E_\parallel V_\parallel, \tag{29}$$

and are analogous to Eqs. (15)–(17).

In the next order, we may take $\bar{f}_i$ to be Maxwellian on the left-hand side of Eq. (23) and decompose the correction $\bar{f}_{i1}$ into an odd and an even piece in $u_\parallel \equiv v_\parallel - V_\parallel$, $\bar{f}_{i1} = F_{i1} + G_{i1}$. To obtain the particle and heat fluxes, we need only the odd piece F_{i1}, which obeys:

$$C_i(F_{i1}) = \sum_j \left[C_{ij}(F_{i1}, f_{j0}) + C_{ij}(f_{i0}, F_{j1}) \right] = u_\parallel \left[A_{i1} + \left(\frac{m_i u^2}{2T} - \frac{5}{2} \right) A_{i2} \right] f_{i0}, \tag{30}$$

where $u \equiv v - V_\parallel$, the sum is taken over all species (including electrons), and

$$A_{i1} \equiv \nabla_\parallel \ln(n_i T_i) - \frac{e_i E_\parallel^i}{T} - \frac{m_i}{T_i} \frac{\partial}{\partial r} \left(D_a \frac{\partial V_\parallel}{\partial r} \right) - \frac{2 m_i D_a}{T_i} \frac{\partial V_\parallel}{\partial r} \frac{\partial \ln n_i}{\partial r}, \tag{31}$$

$$A_{i2} \equiv \nabla_\parallel \ln T_i - \frac{2 m_i D_a}{T_i} \frac{\partial V_\parallel}{\partial r} \frac{\partial \ln T_i}{\partial r}, \tag{32}$$

are thermodynamic forces. Here we have introduced $E_\parallel^i \equiv E_\parallel - (m_i/e_i)(V_\parallel + \bar{V}_E) \cdot \nabla V_\parallel$. Eq. (30) has the form of a Spitzer problem for classical transport along the magnetic field. In fact, if the diffusion coefficient D_a is independent of velocity, Eq. (30) is mathematically entirely equivalent to the multi-ion Spitzer problem governing classical parallel ion transport, which has been solved previously [2]. The only difference is that the usual thermodynamic forces are modified by anomalous diffusion through the terms proportional to D_a. These new terms are of the same order as the classical ones because of the edge order of Eq. (4).

We may use the mathematical equivalence to the classical problem to write down the inverse transport laws

$$R_i = \sum_j \left(l_{11}^{ij} u_{\parallel j} + \tfrac{2}{5} l_{12}^{ij} q_{\parallel j}/p_j \right), \tag{33}$$

$$H_i = \sum_j \left(l_{21}^{ij} u_{\parallel j} + \tfrac{2}{5} l_{22}^{ij} q_{\parallel j}/p_j \right), \tag{34}$$

relating the total parallel friction force R_i acting on the species i

$$R_i \equiv \int \bar{f}_i m u_\parallel \, d^3u = n_i T_i A_{i1},$$

and the heat friction

$$H_i \equiv \int \bar{f}_i m u_\parallel \left(\frac{m_i u^2}{2T} - \frac{5}{2} \right) d^3u = \tfrac{5}{2} n_i T_i A_{i2}, \tag{35}$$

to the particle and heat fluxes, $u_{\parallel i}$ and $q_{\parallel i}$, respectively. The transport coefficients $l_{kl}^{ij} = l_{lk}^{ji}$ are identical to the classical ones [2]. By inverting the system of equations Eqs. (33) and (34), one obtains the particle and heat fluxes as linear combinations of the thermodynamic forces. Alternatively, by solving for R_i and $q_\parallel$ in terms of H_i and $u_{\parallel i}$, transport equations of the type derived by Braginskii [1] and often used in numerical edge computations, are obtained. Thus, when written in this latter form, the usual classical parallel transport laws are modified by the replacement

$$n_j \nabla_\parallel T_j \rightarrow n_j \nabla_\parallel T_j - \frac{2 m_j n_j D_a}{T_j} \frac{\partial V_\parallel}{\partial r} \frac{\partial T_j}{\partial r}, \tag{36}$$

for all j, in the expressions for R_i and $q_{\parallel i}$

For instance, in a hydrogen plasma with heavy impurities, the classical force Eq. (2), modified by the turbulence, becomes

$$R_i = -C_1 \frac{m_i n_i (V_{i\parallel} - V_{z\parallel})}{\tau_{iz}} - C_2 \left(n_i \nabla_\parallel T_i - \frac{2 m_i n_i D_a}{T_i} \frac{\partial V_\parallel}{\partial r} \frac{\partial T_i}{\partial r} \right). \tag{37}$$

More generally, when there are many ion species present, inverting Eq. (34) to obtain $u_{\parallel i}$ and substituting in Eq. (33) gives the force acting on each species as

$$R_i = -\sum_j \left[\alpha_{ij} \frac{m_i n_i (u_{\parallel i} - u_{\parallel j})}{\tau_{ij}} + \beta_{ij} \left(n_j \nabla_\parallel T_j - \frac{2 m_j n_j D_a}{T_j} \frac{\partial V_\parallel}{\partial r} \frac{\partial T_j}{\partial r} \right) \right], \tag{38}$$

where the coefficients α_{ij} and β_{ij} are complicated functions of the masses, densities and charges of all species. Explicit formulae exist in the literature, as well as prescriptions for how to evaluate them numerically [4]. They do not change as a consequence of the edge ordering Eq. (4), although the force itself is modified by the replacement Eq. (36).

A new and unconventional type of thermal force [7] has appeared as the third term in Eqs. (37) and (38). It arises in a way similar to the usual thermal force (the second term, proportional to $\nabla_\parallel T$), i.e. as a consequence of an asymmetry in $u_\parallel$ in the distribution function of the lighter species (in Eq. (37) the hydrogenic ions (i)). For the usual thermal force, the asymmetry arises since the particles travelling in the direction of $\nabla_\parallel T_i$ are colder than the ones moving in the opposite direction. The former are therefore more collisional and push the heavier particles in the direction of $\nabla_\parallel T_i$. In an edge plasma with $\partial V_\parallel / \partial r < 0$ and $\partial T_i / \partial r < 0$, anomalous diffusion transports hot ions with large $V_\parallel$ outwards in the SOL, where ions with $u_\parallel < 0$ therefore tend to be colder and hence more collisional, than the ones with $u_\parallel > 0$. Thus, again, a thermal force arises.

4. Conclusions

In edge plasmas, where the order of Eq. (4) is satisfied, cross field diffusion, whether classical or anomalous, affects parallel ion transport by modifying the parallel force acting between different ion species. In other words, in the presence of anomalous diffusion, the parallel transport is not, in general, purely classical and even if the cross field diffusion is classical, the parallel transport laws assume an unconventional structure.

To make these phenomena explicit, we have adopted the simplest possible models for the cross field transport. Thus, classical cross field diffusion is taken to occur as a consequence of collisions between light main ions and heavy impurities and anomalous transport is described by quasilinear diffusion. It must be emphasized that these simplifications are made solely for the sake of transparency and analytic tractability. The basic point we wish to make, only hinges on the ordering Eq. (4) and is independent of the mechanism underlying the radial transport. The corrections to the classical force are given by Eqs. (21) and (22) in the case of classical diffusion, and in a turbulent, multi-species plasma it is given by the last term in Eq. (38). In both cases, the new force, although acting in the parallel direction, is driven only by radial gradients.

It may be of practical interest for impurity retention in tokamak divertors, that if $\partial V_\parallel / \partial r$ and $\partial T_i / \partial r$ have the same sign, the force proportional to $(\partial V_\parallel / \partial r)(\partial T_i / \partial r)$ opposes the classical one, which otherwise tends to drive impurity ions from the divertor towards the core plasma. In the divertor plasma, the parallel velocity and the temperature usually both have a maximum near the separatrix, falling off towards the outer SOL and towards the private flux region. In this case the new force pushes impurities towards the divertor plates. To assess its effect more accurately requires numerical simulation of the fluid equations in realistic geometry. Without such a numerical calculation it is difficult to establish the actual magnitude of the new terms found in the present work. The fundamental arguments in Section 1 force us to adopt the ordering Eq. (4), which, taken literally, implies that the new terms are of the same order as the conventional ones. However, in a medium-sized tokamak with $L = 10$ m, $D = 0.2\ \mathrm{m^2/s}$, $W = 1$ cm, we have $v_T / L_\parallel \leq D / W^2$ only at quite low temperatures, $T \leq 4$ eV. On the other hand, the new terms in the ion-impurity force may, even if they are relatively small, alter the balance between thermal and frictional forces. In any case, the new results presented in this work are mostly likely to be important in edge plasmas with strongly sheared parallel flows.

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